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# **A Possible Explanation for the “Parallel Tracks” Phenomenon in Low-Mass X-Ray Binaries**

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## **ABSTRACT**

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An explanation is proposed for the observation that in low-mass X-ray binaries the correlation between most observable X-ray spectral and timing parameters on the one hand, and X-ray luminosity on the other, while generally good in a given source on a time scale of hours, is absent both on longer time scales and between sources. This phenomenon, particularly evident in kHz QPO sources, leads to parallel tracks in plots of such parameters vs. luminosity. It is pointed out that where previously proposed explanations require at least two time-variable independent parameters, such as accretion rate through the disk and through a more radial inflow, just one independent variable is in fact sufficient, provided that the systemic response to time variations in this variable has both a prompt and a time-averaged component. A specific scenario is explored in which most observable spectral and timing parameters to first order depend on disk accretion rate normalized by its own long-term average rather than on any individual accretion rate (luminosity, on the contrary, just depends on the total accretion rate). This provides a way in which parameters can be uncorrelated to accretion rate, yet vary in response to variations in accretion rate. Numerical simulations are presented of such a model describing the relation between kHz QPO frequency and X-ray luminosity, which observationally is characterized by a striking pattern of parallel tracks in the frequency vs. luminosity plane both in individual sources and across sources. The model turns out to reproduce the observations remarkably well. Physical interpretations are suggested that would produce such a scenario; particularly promising seems an interpretation involving a radial inflow with a rate that derives through a time-averaging process from the disk accretion rate, and an inner disk radius that depends on the balance between the accretion through the disk and the total luminosity. The consequences of this idea for our understanding of states and tracks in LMXBs are discussed, and the applicability of the idea to black-hole candidates, where the observational situation is more complex, is briefly addressed.

*Subject headings:* accretion, accretion disks — stars: neutron — X-rays: stars — black hole physics

## 1. Introduction

The relation between the X-ray spectral/timing states of low-mass X-ray binaries (LMXBs) and their X-ray luminosity is puzzling. While on short time scales (typically, hours) X-ray spectral and timing parameters vary (stochastically) in a well-correlated fashion, on

longer time scales (typically, days) the X-ray luminosity, particularly in the low-energy band ( $< 6$  keV) appears to vary largely independently from the other parameters. This is seen in the near-Eddington neutron-star Z sources (e.g., Hasinger et al. 1990, Kuulkers et al. 1996, Wijnands et al. 1998, Jonker et al. 2000) as well as in the less luminous neutron-star atoll sources (e.g., van der Klis et al. 1990, Ford et al. 1997). Observations that could be interpreted in terms of a similar phenomenon in black-hole candidates (e.g., Méndez and van der Klis 1997, Rutledge et al. 1999) have recently been provided with solid empirical underpinnings by a study of the black-hole candidate XTE J1550–564 (Homan et al. 2001).

In all these sources, timing properties, in particular the characteristic frequencies of QPO and noise components correlate much better to each other (e.g., Ford and van der Klis 1998), and to measures of X-ray spectral shape such as position in the tracks LMXBs tend to trace out in an X-ray color-color diagram (e.g. Méndez and van der Klis 1999), X-ray colors (Méndez et al. 1999; particularly at higher energy), and sometimes parameters obtained from X-ray spectral fits (Kaaret et al. 1998), than to luminosity (see van der Klis 2000), although weak dependencies of timing and spectral properties on long-term luminosity variations are observed (Kuulkers et al. 1996, Wijnands et al. 1997). (For example, spectral shape in the  $< 6$  keV band is usually somewhat affected by the longer time scale variations, spectral hardness in this band being positively correlated with flux). The time scale on which the correlated short-term variability takes place is hours to days in the Z sources and in atoll sources in luminous states, but it is sometimes longer in black-hole candidates and in atoll sources in low-luminosity states.

So, while on short time scales all properties including luminosity vary in a well-correlated fashion, on longer time scales X-ray luminosity is the odd one out in an otherwise still rather well-correlated set of spectral and timing parameters. When for a given source a plot is made of any spectral or timing parameter vs. X-ray flux, then, due to this phenomenon, the result tends to be a series of parallel tracks, each individual track reflecting the short-term correlated variations at a given epoch, and the offsets between the tracks resulting from the longer-term luminosity variations, combined with observational windowing. Perhaps the most direct measurements of this parallel-track phenomenon have been made using the frequencies of the kilohertz quasi-periodic oscillations (kHz QPOs) in the Z and atoll sources. For this reason, most of the remainder of this paper focuses on the parallel track phenomenon as seen in kHz QPO sources (Fig. 1); I return to the general case in §4.

### 1.1. Parallel tracks and kilohertz QPOs

Kilohertz QPOs are observed in the X-ray flux of some twenty Z and atoll sources and are generally interpreted as being due to the motion of matter within a few stellar radii of the low-magnetic-field neutron star in these systems (van der Klis 2000 for a review). Most models involve tight orbital motion of matter around the star at a preferred radius  $r$  in the inner accretion disk, where  $r = 12 - 25$  km. Because of this proximity of their site of origin to the compact object, kHz QPOs can potentially be used to constrain theories of strong gravity and dense matter.

A discussion of the physics which might determine  $r$  was given by Miller et al. (1998) (see also Miller and Lamb 1993, 1996). In their description,  $r$  is the inner radius of the Keplerian disk, which as long as it is larger than the innermost stable orbit from general relativity, is set by radiation drag on the orbiting matter. For a constant mass inflow, the radius  $r$  increases when the X-ray flux impinging upon the inner edge of the disk increases; for a constant radiation field,  $r$  decreases when the amount of matter accreting in the disk increases. When both mass flow and radiation increase, it is not obvious what will be the effect on  $r$ , but according to Miller et al. (1998) if all radiation is due to accretion through the disk at a rate  $\dot{M}$ , then  $r$  will decrease when the accretion rate increases. Under certain simplifying assumptions,  $r \propto \dot{M}^{3/2}$ . Other proposed kHz QPO models involving the frequency of orbital motion (e.g., Stella and Vietri 1999, Titarchuk et al. 1999, Cui et al. 1998, Cui 2000, Campana 2000, Psaltis and Norman 2000) either adopt the Miller et al. (1998) mechanism, or rely on a magnetosphere to set the inner disk radius.

The dependence of kHz QPO frequency  $\nu$  on X-ray flux  $F_x$  shows the above-described parallel track pattern. Whereas in a given source on short time scales (less than several hrs) a good correlation is observed between  $\nu$  and  $F_x$ , on longer time scales (more than a day or so) the correlation breaks down. On those longer time scales, different flux levels can correspond to approximately similar frequency ranges. Observational windowing has so far prevented a careful study of what happens on intermediate time scales. In a frequency vs. flux plot of a series of observations of a given kHz QPO source, each lasting several hours and separated by intervals of days or longer, a series of parallel tracks is observed, each track corresponding to the  $\nu$ ,  $F_x$  relation of a single observation.

These parallel tracks are perhaps most clearly seen in the transient atoll source 4U 1608–52. In Fig. 1a; Méndez et al. 1999) the parallel tracks are obvious; they are steeply inclined, approximately straight lines. Note how there seems to be a systematic trend in the steepness of these lines: at lower count rate they tend to be steeper. The range in flux over which in a given source identical frequencies are observed can be as large as a factor of 3. Note that there appears to be a tendency for the lines at the highest flux to cover a somewhat higher

frequency range, and vv.

A similar and perhaps related, but more extreme picture is presented by observations of different sources: the same range of QPO frequencies is observed in sources that differ in X-ray luminosity  $L_x$  (defined here as  $4\pi d^2 F_x$  where  $d$  is the distance) by up to more than two orders of magnitude (van der Klis 1997, Ford et al. 2000). In a plot of frequency vs. luminosity covering the ensemble of kHz QPO sources this once again leads to a series of parallel tracks, each “track” now corresponding to a different source (Fig. 1b; Ford et al. 2000). Note, that these tracks are approximately parallel in a plot of frequency vs. the *logarithm* of  $L_x$ , so the same trend of steeper lines at lower luminosity as observed in 4U 1608–52 clearly also applies across the ensemble of sources. (In a semi-logarithmic version of the plot in Fig. 1a, the lines are also more parallel). In both figures it is as if frequency depends not on  $L_x$ , but on the percentage by which  $L_x$  deviates from its average (cf. van der Klis 1999).

## 1.2. Proposed explanations

The question in both cases illustrated in Fig. 1 of course is: if at some epoch in some source frequency depends on luminosity according to some particular relation, then why at some other epoch or in some other source is there a very different frequency-luminosity relation, where very similar frequencies are attained at very different luminosities? Any explanation that succeeds in obtaining the *same* frequency at quite *different* luminosity would at first sight seem to defy itself, as it would at the same time remove the observed strong dependence of frequency on luminosity at any given epoch. As explained below, up to now the answer typically has been that at least two independently varying free parameters must characterize the problem.

The phenomenology clearly suggests that a single quantity, variable on time scales of less than a few hours, dominates kHz QPO frequency and most other spectral and timing parameters, but not the observed X-ray luminosity. Usually, this parameter has been inferred to be the mass accretion rate  $\dot{M}$ , sometimes parametrized by the curve length  $S$  measured along the track the source traces out on these time scales in the X-ray color-color diagram (§1; Hasinger and van der Klis 1989, van der Klis 1995 for a review). A problem with this interpretation is that the X-ray luminosity, whose energy ultimately derives from this mass accretion, is on longer time scales not then well-correlated to inferred  $\dot{M}$ . Another parameter, varying on time scales of more than a day or so, is therefore required to explain the slow luminosity variations that do not affect the kHz QPO frequency, i.e., the horizontal offset between the parallel lines in a given kHz QPO source (Fig. 1a). Slow variations in

the accretion geometry leading to changes in X-ray beaming (e.g., van der Klis 1995) or in the ratio between matter accreting in the disk and through some other channel such as a more radial inflow (e.g., Wijnands et al. 1996, Kaaret et al. 1998) energy loss modes in unobservable spectral bands (e.g., van der Klis et al. 1995) and mechanical energy in jets (e.g., Méndez et al. 1999), have all been suggested to provide this additional parameter, but in the absence of a specific mechanism to make these parameters vary independently from inferred  $\dot{M}$ , the true origin of the slow luminosity variations has been difficult to pin down (see Ford et al. 2000 for a discussion of these various possibilities).

Yet another parameter, different from source to source, is required in this picture to explain the parallel lines spanning two orders of magnitude in  $L_x$  accross the ensemble of kHz QPO sources (Fig. 1*b*). It has been proposed that this parameter is the neutron-star magnetic-field-strength, which should then be correlated to average source luminosity rather tightly (White and Zhang 1997). Systematic differences between sources with respect to one of the slowly varying parameters mentioned above could perhaps explain Fig. 1*b*, but it has remained unclear which parameter could fulfill all requirements (e.g., Ford et al. 2000). All proposals to date effectively require the existence of at least two independent time-variable parameters, and no compelling description was given of a mechanism that could make such a second time-variable parameter vary on its own accord independently of  $\dot{M}$ .

It is the purpose of this paper to point out that the ensemble of observations described above can in fact be explained in terms of just a single time-variable independent parameter, varying on time scales of hours. This can be accomplished by exploiting the observed difference in time scale between correlated and uncorrelated behavior. The crucial requirement is that there exists both a prompt and a time-averaged component in the systemic response to the variations in the governing parameter. The basic class of scenarios that could accomplish this is outlined in §2, and a specific proposal is made there, namely that the governing parameter is the mass accretion rate through the disk. In §3 it is quantitatively investigated to what extent such a model could explain the observational facts presented above. Numerical simulations of the predicted luminosity, kHz QPO frequency relation in a representative example of such a scenario are described there. In §4 I discuss the results, and the extent to which the scenario might be applicable to the parallel-track phenomena observed in neutron-star and black-hole low-mass X-ray binaries in general, as well as some of the wider implications of the proposed explanation for the interpretation of states and tracks in LMXBs.

## 2. Scenario

If the kHz QPO frequency  $\nu$  depends on some parameter  $x(t)$  which varies on time scales of less than several hours, and the X-ray luminosity  $L_x$  depends partly on  $x(t)$  and for another part on some running average of  $x$  such as  $\langle x \rangle(t) = \int_{-\infty}^{+\infty} w(t' - t)x(t')dt'$ , where  $w(t)$  is a weight function with integral 1 that is non-zero on interval  $[t_1, t_2]$  spanning between several hours and days, then this could explain the parallel tracks in a given kHz QPO source. The correlation between  $\nu$  and  $L_x$  on time scales of hours would arise because both quantities depend on  $x(t)$ , the decorrelation on longer time scales because the slowly varying  $\langle x \rangle(t)$  affects  $L_x$ , but not  $\nu$ .

However, such a description would for plausible prescriptions for  $\langle x \rangle$  have trouble to explain that a similar range of frequencies is seen in 4U 1608–52 for fluxes different by a factor of 3, and could certainly not explain that the same frequencies are observed over several orders of magnitude in luminosity across sources. For  $L_x$  to be very different,  $x$ , and hence  $\nu$  would have to be very different, contrary to what is observed. Also, in general the lines would not be parallel in  $\nu$  vs.  $\log L_x$  space.

With an additional twist, such a scheme *can* explain both parallel-track phenomena. Suppose, for example, that  $\nu$  is proportional to  $\eta \equiv x/\langle x \rangle$ , but  $L_x$  is proportional to  $x$  only. Then, when  $x$  increases,  $\nu$  and  $L_x$  immediately increase in correlation with it, but if  $x$  remains the same for a while, then on a time scale of more than several hours  $\nu$  would gradually drift to the value it has when  $\eta = x/\langle x \rangle = 1$ . In such a picture a 100 times more luminous source can still show the same QPO frequency as its weaker kin because not only  $x$  but also  $\langle x \rangle$  is approximately 100 times higher, so that  $\eta = x/\langle x \rangle$  is still approximately the same. Note also that in such a scheme the same fractional variation in  $x$  tends to cause a similar absolute change in  $\nu$ , so that the lines are parallel in  $\nu$ ,  $\log L_x$  space, as observed. Several similar prescriptions for the dependence of  $\nu$  and  $L_x$  on  $x$  and  $\langle x \rangle$  are possible that preserve the desirable qualities of this description.

An attractive interpretation retaining the flavor of this basic idea that would work well within the framework of a model where the inner radius  $r$  of the disk, and hence  $\nu$ , is determined by some balance between accretion rate through the disk and radiative stresses (§1.1), would be one where  $\nu$  depends on the ratio between instantaneous accretion rate through the disk  $\dot{M}_d(t)$  varying on short time scales, and  $L_x$ :

$$\nu(t) \propto \left( \frac{\dot{M}_d(t)}{L_x(t)} \right)^\beta, \quad \beta > 0 \quad (1a)$$

and the luminosity has both an immediate response to  $\dot{M}_d$  and a filtered one:

$$L_x(t) \propto \dot{M}_d(t) + \alpha \langle \dot{M}_d \rangle(t), \quad \alpha > 0 \quad (1b)$$

So, in this model  $x(t)$  is identified with the accretion rate  $\dot{M}_d(t)$  through the disk,  $\nu$  depends on  $x/(x + \alpha \langle x \rangle)$ , and  $L_x$  on  $x + \alpha \langle x \rangle$ . Note that  $x/(x + \alpha \langle x \rangle) = (1 + \alpha/\eta)^{-1}$  where  $\eta \equiv \dot{M}_d/\langle \dot{M}_d \rangle$ , so  $\nu$  still only depends on the ratio  $\eta$  as in the simpler prescription above. For this reason, the model retains the basic qualities of this simpler prescription. At epochs where  $\eta < 1$ , i.e.,  $\langle \dot{M}_d \rangle > \dot{M}_d$  too much radiation is produced for the amount of matter in the disk (set by  $\dot{M}_d$ ) so  $r$  is large and hence  $\nu$  low, and vv. If  $\dot{M}_d$  remains the same for a while, then  $\langle \dot{M}_d \rangle$  approaches  $\dot{M}_d$ , i.e.,  $\eta \approx 1$ , and  $r$  and hence  $\nu$  drift to their “equilibrium” value, no matter what the actual value of  $\dot{M}_d$  is. The lines tend to be parallel in  $\nu$ ,  $\log L_x$  space as, depending only on the extent to which  $\langle \dot{M}_d \rangle$  has converged to  $\dot{M}_d$ , the same fractional change in  $\dot{M}_d$  and hence  $L_x$  tends to produce the same linear change in  $\nu$ . Note that no other independently time-varying parameter than  $\dot{M}_d$  is required now to explain the parallel-tracks phenomenon.

The physical question is: what is the nature of the slowly varying component in the response of luminosity to accretion rate, the term  $\alpha \langle \dot{M}_d \rangle(t)$ ? Physical implementations could be imagined using for example a mechanism for the storage and slow release of energy in the neutron star, such as through deposition and burning of nuclear fuel, energy storage in the crust or in the disk/star boundary (Inogamov and Sunyaev 1999, Popham and Sunyaev 2001). Another possibility would be energy storage in the neutron-star spin, with energy feeding back into the disk by magnetic or gravitational interaction (cf. Friedhorsky 1986). Alternatively, one could think of a mechanism where matter flowing in radially provides a second channel of accretion whose  $\dot{M}$  is derived from some running average over  $\dot{M}_d$ , for example because more matter in the disk leads to more matter blowing off it to feed the radial flow, or because disk and radial flow are both fed by the same external flow.

Note from these examples that the filtered response can technically be either causal (e.g., burning rate depends on amount of fuel accreted in the past), or acausal (e.g., matter blown from disk reaches neutron star before disk flow); the response may even be symmetric. The basic feature of the model scenario discussed here is that more accretion through the disk produces more flux not only promptly, but also through some time-averaged response, and (only) when this filtered response is at the level it would have when the accretion were constant, do the effects on the QPO frequency cancel.



### 3. Simulations

For definiteness, assume the following description for the dependence of frequency on accretion rate and X-ray luminosity:

$$\nu = \nu_0 \left( \frac{\epsilon \dot{M}_d(t) c^2}{L_x(t)} \right)^\beta = \nu_0 \left( \frac{\dot{M}_d(t)}{\dot{M}_d(t) + \alpha \langle \dot{M}_d \rangle(t)} \right)^\beta \quad (2a)$$

where the luminosity is simply  $L_x(t)/c^2 = \epsilon(\dot{M}_d(t) + \dot{M}_{rad}(t))$  with  $\epsilon$  the accretion mass-to-energy conversion efficiency, and where the radial accretion rate is related to the disk accretion rate via  $\dot{M}_{rad}(t) = \alpha \langle \dot{M}_d \rangle(t)$ . So, in this description  $\eta \equiv \dot{M}_d / \langle \dot{M}_d \rangle = \alpha \dot{M}_d / \dot{M}_{rad}$ . Frequency  $\nu$  only depends on  $\eta \propto \dot{M}_d / \dot{M}_{rad}$  and  $L_x$  on  $\dot{M}_d + \dot{M}_{rad}$ . The filtered response  $\langle \dot{M}_d \rangle$  is given by an integral over the future disk accretion rate:

$$\langle \dot{M}_d \rangle(t) = \frac{1}{\tau} \int_t^\infty e^{-(t'-t)/\tau} \dot{M}_d(t') dt'. \quad (2b)$$

Results of numerical simulation of this description with a radial-to-disk accretion ratio  $\alpha$  of 1/3, filter time scale  $\tau = 2$  days and  $\beta = 2$  are shown in Fig. 2, where a second-order red-noise signal (random walk) was used for  $\dot{M}_d(t)$ , and 12 days of data were plotted in intervals of 8 hrs separated by gaps of 16 hrs to simulate typical observational windowing. Note that these choices for  $\tau$  and windowing time scales are illustrative; the scenario will work for a variety of time scales.

Comparing Fig. 2 to Fig. 1a, clearly the result of this simulation is rather similar to observation. Not only do the parallel lines show up, also the steepening of the tracks towards lower luminosity is reproduced, as well as the tendency for lines at higher count rates to cover higher frequency ranges. As explained in §2, the steepening effect is a consequence of the fact that frequency depends only on  $\eta$ , so that same fractional change in  $\dot{M}_d$  tends to produce the same linear change in  $\nu$ . The other effect arises because the highest count rates tend to be attained at the highest disk accretion rates  $\dot{M}_d$ . Necessarily, these tend to be followed by intervals of lower  $\dot{M}_d$ , so that  $\langle \dot{M}_d \rangle$  tends to be lower than  $\dot{M}_d$  at those epochs, i.e.,  $\eta > 1$ , which explains why relatively higher frequencies often occur there. At low count rates the opposite applies.

Fig. 3 shows the result of the simulation of four sources differing in average  $\dot{M}_d$  but showing a similar fractional amplitude of  $\dot{M}_d$  variations. Each source was simulated for 5 days, during which it varied randomly in luminosity by a factor of 2. Comparing Fig. 3 to Fig. 1b, again the result of the simulation is similar to what is observed. In particular, the simulated tracks distributed over a factor  $\sim 100$  in  $L_x$ , are approximately parallel in  $\nu$ ,  $\log L_x$  space.

Very similar results were obtained from a simulation where the filtered response was representative of one due to nuclear burning of accreted material rather than of radial accretion. In that case,  $L_x/c^2 = \epsilon_{acc}\dot{M}_d(t) + \epsilon_{nuc}\langle\dot{M}_d\rangle(t)$  with  $\epsilon_{acc}$  and  $\epsilon_{nuc}$ , respectively, the accretion and the nuclear burning mass-to-energy conversion efficiencies, typically 0.1 and 0.003, and  $\langle\dot{M}_d\rangle(t)$  now given by a *past* integral over  $\dot{M}_d(t)$ . However, in order for the observed steep dependencies of  $\nu$  on  $L_x$  to be reproduced in those simulations, a large value of  $\beta$  between 10 and 15 had to be used. This is because of the small value of  $\alpha = \epsilon_{nuc}/\epsilon_{acc} = 0.03$  appropriate to that physical example. The work of Miller and Lamb (1993, 1996) suggests that lower values of  $\beta$  such as the value of 2 used in the first example are more appropriate.

The velocity of a radial flow is expected to be much higher than the radial component of the motion of the matter accreting in the disk. This is expressed in Eq. 2b by making  $\dot{M}_{rad}$  reflect the *future* evolution of  $\dot{M}_d$ . Therefore, it might seem surprising that the variations in  $\dot{M}_{rad}$  would be a low-pass filtered version of, i.e., slower than, those in  $\dot{M}_d$ . However, note that the disk is expected to feed the radial flow over an annulus covering a range of radii. The density fluctuations in the disk would contribute to the radial flow for the full time they take to cross this ‘feeding annulus’, i.e., for much longer than they take to cross the inner disk edge and accrete in the end. Alternatively, the short-term fluctuations in  $\dot{M}_d$  could form at small radii in the disk, within the feeding annulus. In both cases  $\dot{M}_{rad}$  would be a smoothed version of  $\dot{M}_d$ , which is all that is required to make the scenario work.

#### 4. Discussion

The parallel tracks observed in kHz QPO sources in plots of QPO frequency vs. logarithmic X-ray luminosity within a given source as well as across sources (Fig. 1) can be explained in terms of just one time-variable independent parameter  $x(t)$ , provided that there is both a prompt ( $< \text{hrs}$ ) and a filtered ( $> \text{a day}$ ) systemic response to the time variations in this parameter. Numerical simulations of a version of this scenario where QPO frequency depends on the balance between X-ray luminosity and accretion rate via the accretion disk, and luminosity depends on this accretion rate, both promptly and filtered by some averaging mechanism, show a remarkable similarity to what is actually observed.

The basic prediction of this entire class of models is that it should be possible to reconstruct QPO frequency from the time variations of the governing parameter  $x$  using a model involving some running average of  $x$ . In the example simulated,  $x$  is identified with the accretion rate through the disk  $\dot{M}_d$  and  $L_x$  is  $\dot{M}_d + \alpha\langle\dot{M}_d\rangle$ , where  $\langle\dot{M}_d\rangle$  is  $\dot{M}_d$  convolved with a response function with characteristic time scale  $\tau$ , so that we are faced with a deconvolution problem which might in principle be solved using time series of simultaneous measurements

of instantaneous  $L_x$  and QPO frequency long enough to cover a sufficient number of correlation time scales  $\tau$ , and dense enough to sample the short time scale variations well. Perhaps this can be accomplished by combining RXTE PCA and ASM observations, although PCA observations extending over several weeks would be much preferable.

For models of the general type expressed in Eqs. 1, both the long-term and short-term variations in  $L_x$  are caused by variations in  $\dot{M}_d$ ; the changes in kHz QPO frequency and presumably in the other frequencies associated with the short-term  $L_x$  variations arise from changes in the inner disk radius  $r$ , which in turn are related to changes in the ratio  $\eta \equiv \dot{M}_d / \langle \dot{M}_d \rangle$  between instantaneous and long-term average  $\dot{M}_d$ . Note, that while the changes in X-ray spectral parameters and QPO amplitudes are also affected by changes in this ratio, this occurs not necessarily exclusively through the effect  $\eta$  has on  $r$ . For the example simulated, if both disk and radial accretion contribute to  $L_x$  with a different spectral shape, then the spectrum would be more similar if the ratio of their rates is similar. If there are radiative-transfer effects of the radial flow on spectrum or QPO amplitudes, these would be stronger when the radial flow is denser; i.e., these effects depend on  $\langle \dot{M}_d \rangle$  instead of on  $\eta$ .

So, in this scenario, the elusive time-variable parameter (often dubbed “inferred  $\dot{M}$ ” in the literature) that to first order determines all QPO frequencies and a number of other timing and spectral parameters (but not  $L_x$ ) turns out to be  $\eta$ , the disk accretion rate normalized by its own long-term average, rather than any straight accretion rate. Curve length  $S$  along the track of a source in the color-color diagram is in this picture a measure of  $\eta$ . Depending on the precise nature of the filtered response  $\langle \dot{M}_d \rangle$  (e.g., nuclear burning or radial accretion),  $L_x$  may or may not be a pure measure of the total accretion rate  $\dot{M}$ , but even if it is, as in the model expressed in Eqs. 2, neither  $L_x$  nor total  $\dot{M}$  are directly correlated with  $\eta$  and hence with all the associated observable parameters. Although the short-term variations in  $\eta$ ,  $r$  and associated observable parameters such as  $S$  and  $\nu$ , as well as those in  $L_x$  do indeed result directly from short-term variations in the disk accretion rate  $\dot{M}_d$ , the value of  $\eta$  does not follow from the actual value of  $\dot{M}_d$  or  $L_x$ , nor from that of any other instantaneous accretion rate such as total  $\dot{M}$ .

This leads to several further predictions. Two observations characterized by different  $L_x$  but the same  $\nu$  in this description find the source with a disk that has the same  $r$  and hence  $\eta$ , but different  $\dot{M}_d$  and  $\langle \dot{M}_d \rangle$ . No strong decrease of QPO fractional amplitude with luminosity is predicted as would be the case if the luminosity variations would be dominated by an extra source of X-rays unrelated to the QPOs — this seems to be borne out by observations (Méndez et al. 2000). Which spectral variations are seen for luminosity variations along or across parallel lines depends on how variations in respectively  $\eta$  and  $\dot{M}$  affect the spectrum. Motion from one parallel track to the other in the  $L_x$ ,  $\nu$  diagram could happen along any

trajectory depending on the  $\dot{M}_d$  record and the precise nature of the filtered response, but no sudden jumps in the frequency-luminosity plane connecting branches far apart would be expected. Note that *each* correlated quantity is predicted to depend to first order on  $\eta$ ; two QPO frequencies, one depending on  $\eta$  and another on  $\dot{M}_d$  would not in the long run remain well correlated to each other.

The scenario discussed in this paper does not aim to explain the *differences* with respect to spectral/timing properties observed as a function of luminosity, which of course exist. Clearly, even if the basic idea is correct that disk accretion rate normalized by its own long-term average,  $\eta = \dot{M}_d / \langle \dot{M}_d \rangle$ , to first order determines most of the phenomenology either because it sets inner disk radius  $r$  or directly, it is not difficult to think of mechanisms that could explain subtle differences in phenomenology at similar  $r$  as a function of modest changes in  $L_x$  such as described by, e.g., Kuulkers et al. (1996), and more obvious differences such as those between Z and atoll sources for large  $L_x$  differences (Hasinger and van der Klis 1989). It is beyond the scope of this paper to speculate on the precise nature of such mechanisms.

#### 4.1. Black holes

Finally, it is of interest to consider to what extent this type of scenario could be of relevance to black-hole candidates. In those sources, a similar type of correlation on short and decorrelation on longer time scales is sometimes observed between  $L_x$  on the one hand and spectral hardness and timing parameters on the other (§1). In the black-hole candidate XTE J1550–564, parallel tracks are in fact observed in an  $L_x$  vs. spectral hardness diagram, with similar hardness and QPO variations occurring along tracks in a hardness vs. count rate diagram at very different luminosity (Homan et al. 2001, who also discuss the possibility that  $r$  varies more or less independently from  $L_x$ ). Perhaps a similar mechanism such as that explaining the similar phenomenology in the neutron star systems underlies this behavior, although matters are complicated by the absence of a solid surface allowing (but not requiring) matter to accrete without producing much radiation (e.g., Esin, McClintock and Narayan 1997), so that the radiative stresses determining  $r$  are harder to predict from the accretion flows.

Reports of “hysteresis” in the flux from black-hole candidates (Miyamoto et al. 1995, Smith, Heindl and Swank 2001; these latter authors discuss their results in terms of two independent accretion flows), with a tendency towards hard spectra when the count rate rises and soft spectra when it falls, suggest that a disk plus radial flow pattern related by a filtered response as described here could play a role in these systems: if the count rate is dominated by the disk accretion rate, and spectral hardness is representative of inner disk

radius then this is exactly what would be expected from the model described in §3: for a long-term dropping count rate  $\eta > 1$  so  $r$  is small and the spectrum soft, and vv. This means that parallel tracks could be produced in the way explained in this paper.

Different from the neutron stars, where there are usually no cross-tracks connecting the parallel tracks, the parallel tracks in XTE J1550–564 are connected together in what was interpreted to be a comb-like pattern (Homan et al. 2001). The very quiet, soft (“high”) state the source is in when it is on the connecting track (the back of the comb) may hence be unique to black holes, perhaps because it is associated to a flow mode where the matter that accretes radially flows into the hole without producing much radiation, so that the radiative stress on the inner edge of the disk flow is low and hence  $r$  small and insensitive to small variations in the accretion flows. The parallel tracks (the teeth of the comb) on which the other black-hole states are observed (“very high”, “intermediate” and “low” states), may correspond to a mode where, perhaps due to interaction with a jet, the radial flow does produce considerable radiation so that  $r$  is larger and sensitively dependent on the accretion flows, as in neutron stars. This scenario would require the radial flow to produce considerable radiation with low angular momentum at radii smaller than  $r$ ; it is unclear if there is a mechanism by which this could be accomplished.

## 5. Conclusion

The scenario described in this paper can, for the first time, explain how it is possible that kHz QPO frequency and several other timing and spectral parameters are the same in sources differing by more than two orders of magnitude in  $L_x$ , and in a given source for  $L_x$  levels different by a factor of several, even when those properties in any given source at any given instant do themselves strongly depend on  $L_x$ . The proposed solution is that such properties to first order depend on  $\eta$ , disk accretion rate normalized by its own long-term average. This at the same time removes the need for additional independent, slowly varying or fixed source-dependent parameters affecting the phenomenology, and provides a way in which most of the observed short-term variations can arise as a consequence of short-term accretion rate variations while preserving the expected strong dependence of luminosity on total accretion rate.

The physical mechanism providing the long-term averaged response to the disk accretion rate variations required in this scenario can be the accretion of matter flowing off the disk vertically and accreting radially, in combination with a mechanism to make the inner disk radius dependent on the ratio of luminosity to disk accretion rate. The radiation produced by this radial accretion flow in the inner disk region is crucial in that case in determining

much of the phenomenology. If in black holes, contrary to neutron stars, there is a mode for this radial flow to accrete while producing little radiation (and another where it produces considerable radiation), then this additional option available to black holes might explain the more complicated phenomenology of those sources as compared to neutron stars.

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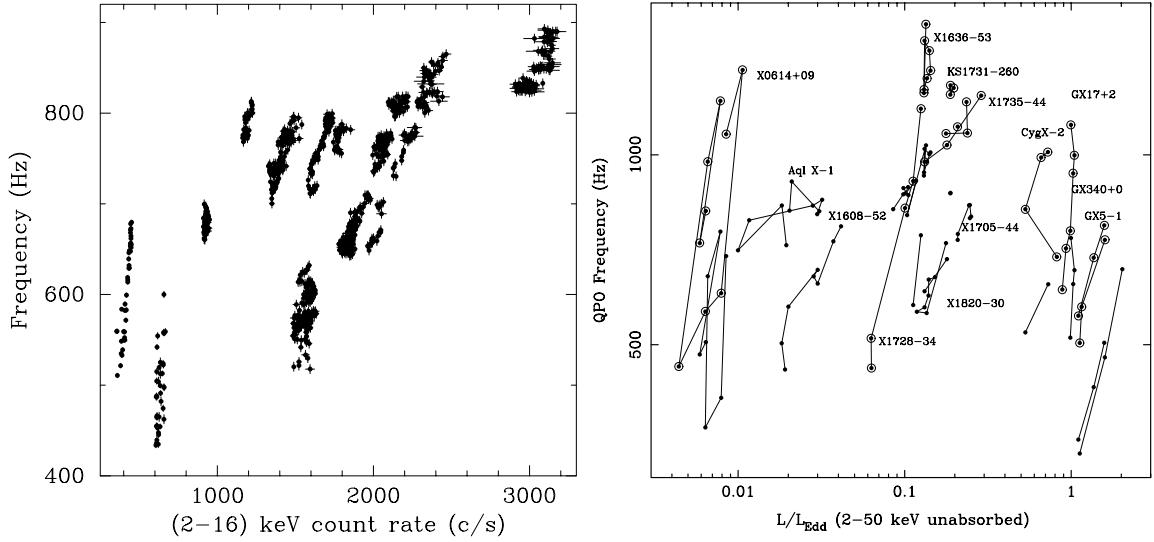


Fig. 1.— a. (*left*): Lower kHz QPO frequency vs. count rate in 4U 1608–52 showing the “parallel lines” phenomenon as it occurs in a single source. After Méndez et al. (1999). b. (*right*): Kilohertz QPO frequencies vs. X-ray luminosity in 13 different sources showing the parallel lines phenomenon across sources. After Ford et al. (2000).



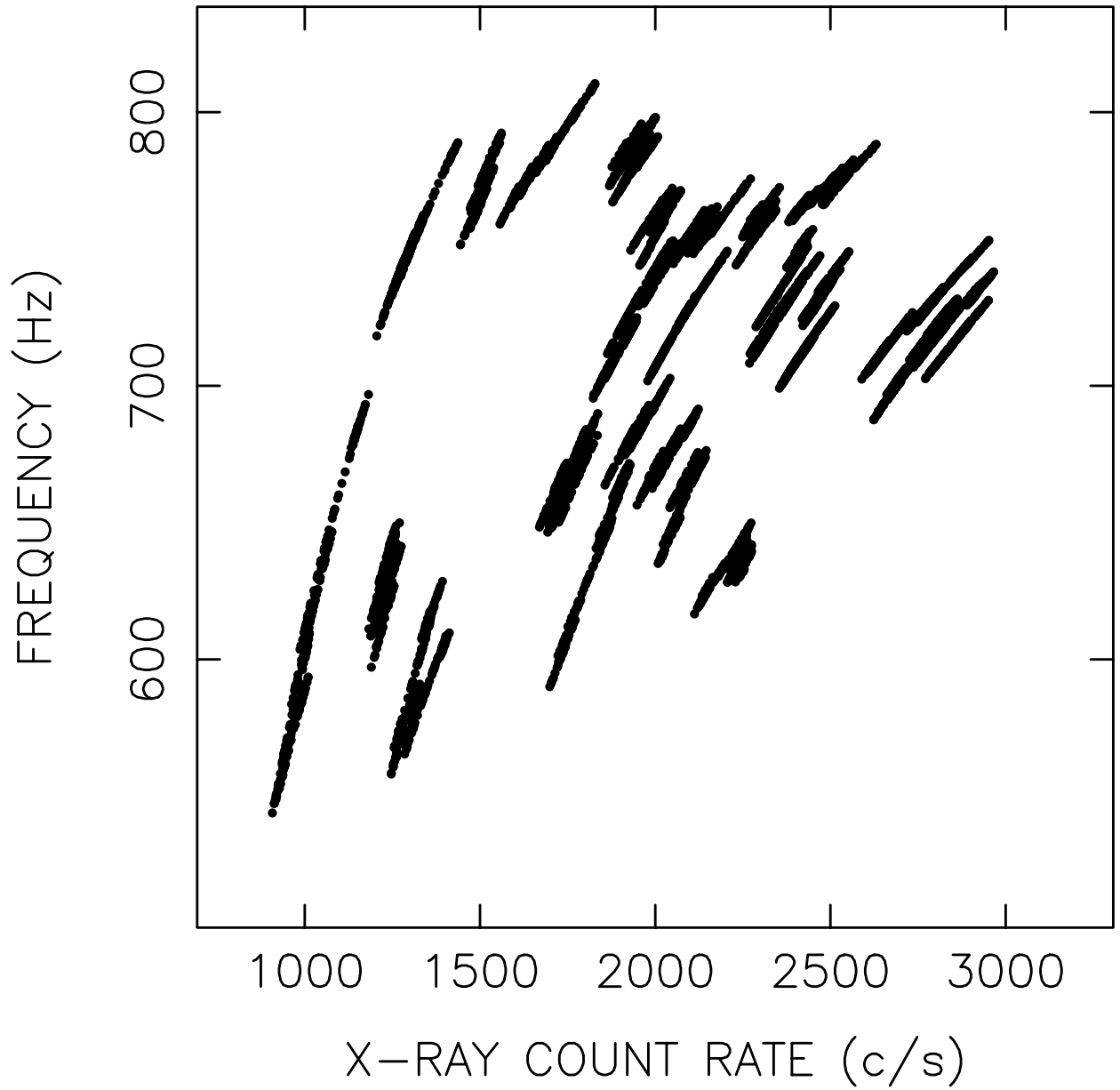


Fig. 2.— Parallel lines in the frequency vs. count rate diagram produced by the model described in §3. Data gaps of 16 hours were introduced in the 12-day simulated time series to mimick typical observational windowing.

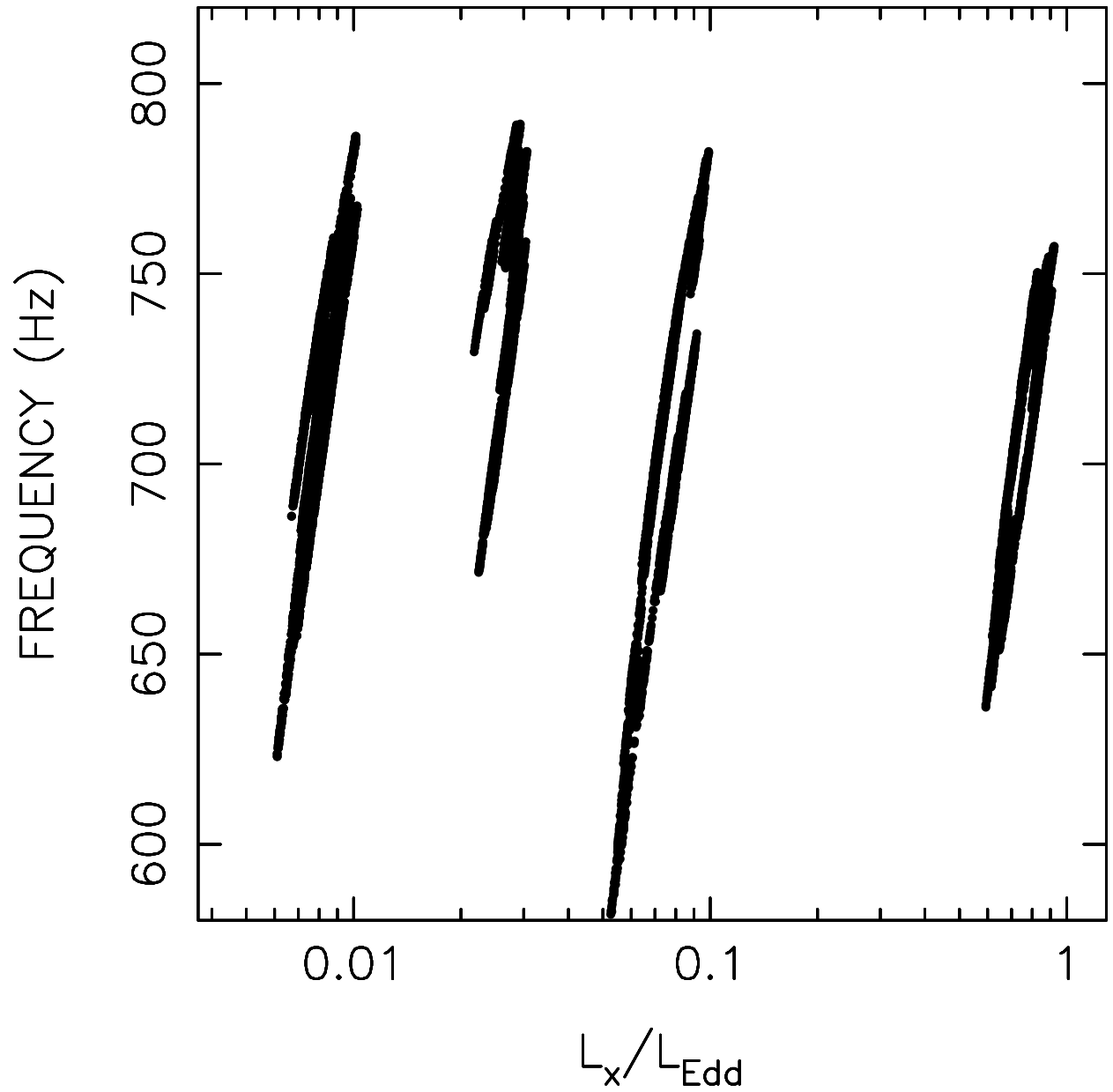


Fig. 3.— Parallel lines in the frequency vs. X-ray luminosity diagram for 4 different simulated sources produced by the model described in §3. Each source was followed for 5 days, during which it varied randomly in luminosity by a factor of 2.